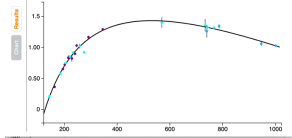
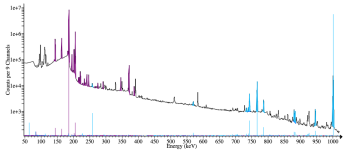
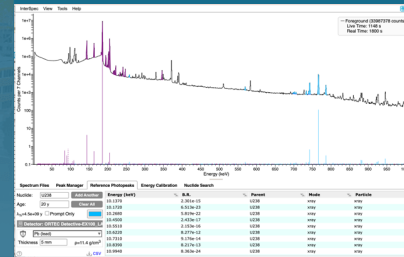




Peak-Based Relative Efficiency analysis in InterSpec



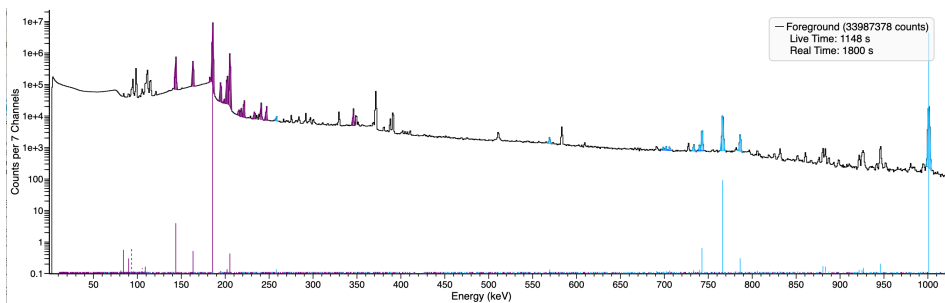
Relative activities and mass fractions

Nuclide	Rel. Act.	Mass Frac.	Uncert.
U235	3.376E7	51.82%	26.4%
U238	4882151	48.17%	25.6%

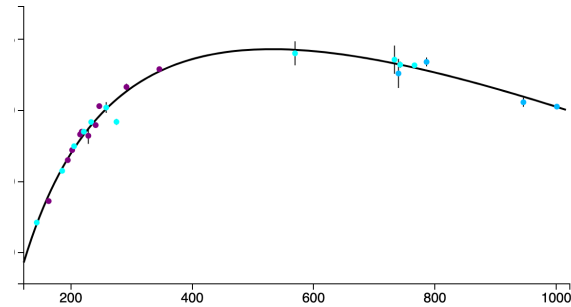
Will Johnson

20220922

Photopeak Detection Efficiency



$$C * \frac{\text{Peak Area}}{\text{Act.} \times \text{B.R.}}$$



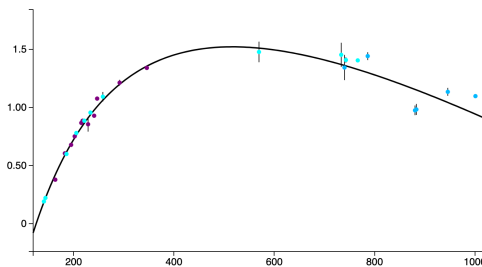
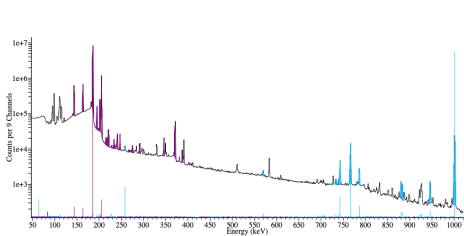
- The peak area divided by the branching ratio for that energy and the nuclides activity, gives an efficiency of detection, as a function of energy.
 - The effects of efficiency include: shielding, self-attenuation, detector efficiency, and distance
- Above the x-ray absorption edges (e.g., above 120 keV), we expect efficiency to be a smooth function of energy
 - The efficiency curve will be a product of the detector efficiency, attenuation from shielding and the air, and geometric effects – all of which are reasonably smooth functions (above x-rays)



Relative Efficiency Curve Fitting



- If we are willing to give-up knowing the absolute normalization of the efficiency curve, and we are okay just assuming a functional form for the shape of the curve:
 - We can then use the known branching ratio of gammas, and the detected peak areas to fit for the *relative activities* of nuclides present.
 - We also fit for the *relative efficiency* curve shape - but we don't actually care about this, beyond checking data is consistent with it
 - Lets assume a relative efficiency curve of form: $y(x) = C_0 + C_1 \ln^1(x) + C_2 \ln^2(x) + C_4 \ln^3(x)$



$$y(x) = 23.17 - 17.54 \times \ln(x) + 3.95 \times \ln^2(x) - 0.27 \times \ln^3(x)$$



Relative activities and mass fractions

Nuclide	Rel. Act.	Mass Frac.	Uncert.
U235	3.376E7	51.82%	26.4%
U238	4882151	48.17%	25.6%

Mass and Activity Ratios.

Nuclides	Mass Ratio	Activity Ratio
U238/U235	0.9296 ± 0.0108	0.1446 ± 0.0697
U235/U238	1.076 ± 21.42	6.915 ± 3.332

Relative Efficiency Analysis



Does **not** require:

- Detector efficiency knowledge
- Source composition or geometry knowledge
- Shielding/attenuation knowledge
- Detector to source distance knowledge

Does require:

- Photopeaks at enough energies to sufficiently fit the relative efficiency curve
- The source/shielding to be homogenous
 - E.g., nuclides must be co-located, source object is of uniform material composition (but can be any shape), and shielding is consistent (no holes in shielding material, etc)
- You to identify peaks, and attribute sources to them

The analysis yields:

- Ratio of activities, which for uranium and plutonium gives you enrichment
- Nuclide ages (when gamma spectra for a nuclide evolves with age, ex Pu241)

Useful Resources

- *Relative Efficiency Curves Demystified*, by Mike Enghauser SNL
 - A great introduction to the topic, focusing on uranium enrichment determination
 - <https://www.osti.gov/servlets/purl/1399186>
- FRMAC Gamma Spectroscopist Knowledge Guide, by Mike Enghauser SNL
 - See section 14. This entire document is a great guide for practical gamma spectroscopy
 - <https://www.osti.gov/biblio/1763003>
- *Application Guide to Gamma-Ray Isotopic Analysis Using the FRAM Software*, LANL
 - This document contains a thorough description of the Relative Efficiency analysis methodology, especially as its related to Uranium and Plutonium analysis.
 - <https://www.lanl.gov/orgs/n/n1/appnotes/LA-14018-M.pdf>



Relative Efficiency Analysis in InterSpec

Starting with v1.0.11, InterSpec contains two different tools for performing Relative Efficiency analysis

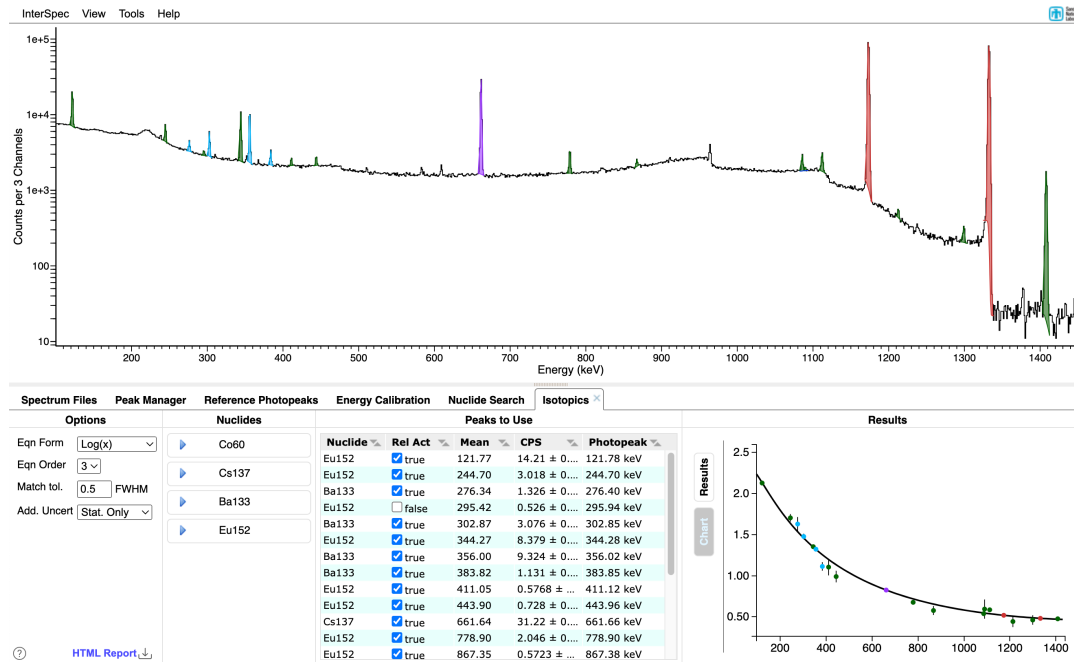
- “Isotopics from peaks”: uses peaks you manually fit, and nuclide identifications you make, to fit for relative activities of the nuclides

The screenshot displays the InterSpec software interface for relative efficiency analysis. The main window is titled "Isotopics" and contains several panels:

- Options Panel:** Includes "Eqn Form" (Log(x)), "Eqn Order" (3), "Match tol." (0.5), and "Add. Uncert" (Stat. Only).
- Reference Photopeaks Panel:** Lists nuclides: Co60, Cs137, Ba133, and Eu152.
- Table:** A table with columns: Nuclide, Rel Act, Mean, CPS, and Photopeak. It lists data for Eu152, Ba133, and Cs137.
- Results Plot:** A graph showing relative efficiency (y-axis, 0.50 to 2.5) versus energy (x-axis, 200 to 1400 keV). Data points are shown as colored diamonds, and a black curve represents the fit.
- Relative Act. Isotopics Window:** A separate window showing a spectrum with fitted peaks. It includes configuration options for "Eqn Type" (Log(y)), "Eqn Order" (1), "FWHM Form" (sqrt(AD + A^2)), and "Energy Ranges" (Lower Energy: 614 keV, Upper Energy: 671 keV).

- “Isotopics by nuclides” (*still under development!*) You provide a list of nuclides, and energy ranges you want to use, and InterSpec fits the peaks and everything else
 - Use of this tool not covered in this presentation

Isotopics from Peaks Tool



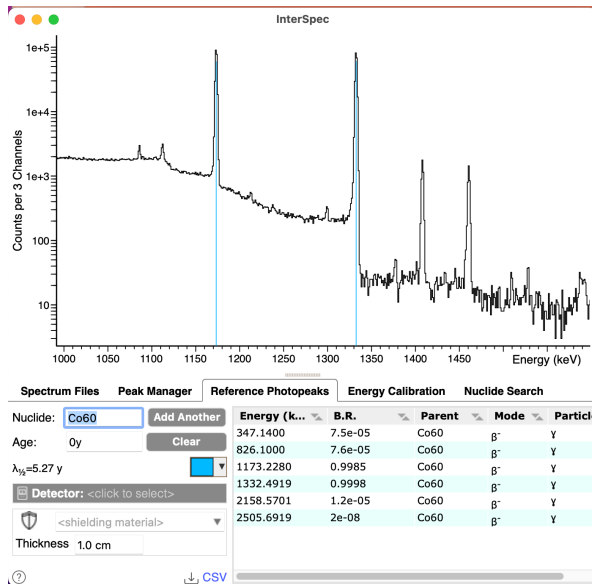
The "Isotopics from Peaks" is similar to the "Activity/Shielding Fit" tool, in that to use it, you must first fit for the peaks of interest in the spectrum, and associate nuclides with them

Fitting peaks reminder

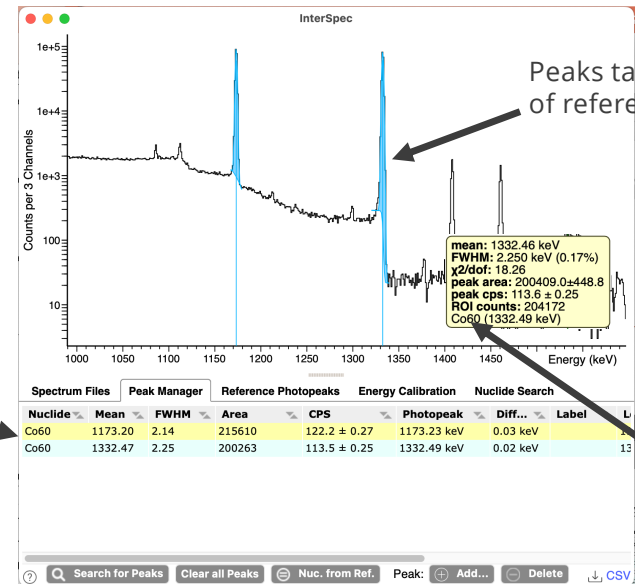


When you fit a peak, you *usually* also want to associate a nuclide with that peak.

The easiest way to do that, is to show reference photopeak lines **before** fitting peaks



Step 1: show Reference Photopeak



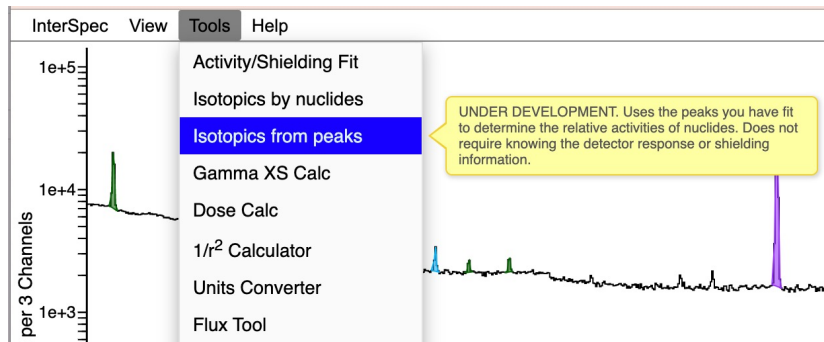
Nuclide shows in table

Nuclide shows in mouse-over info

Step 2: double-click on spectrum to fit peaks

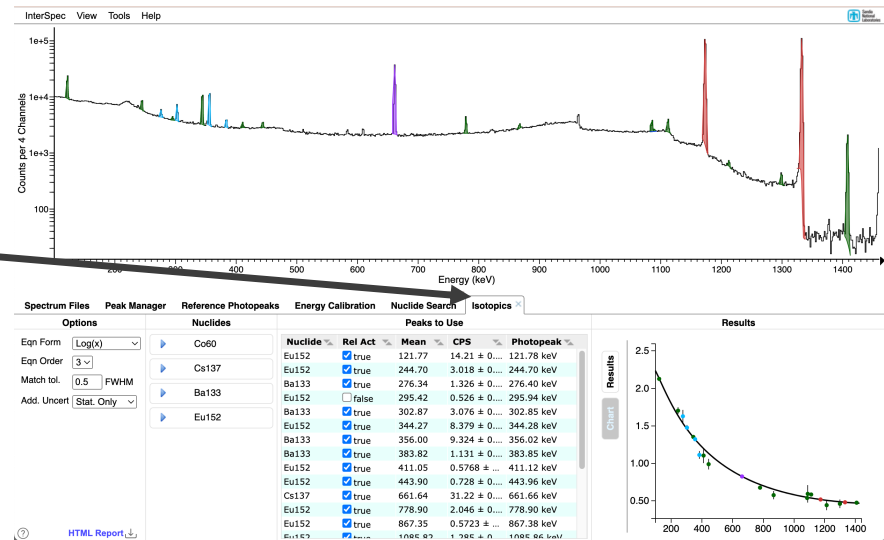
(you can also assign nuclides by editing the peak table, or by right-clicking on the peak)

Isotopics from peaks

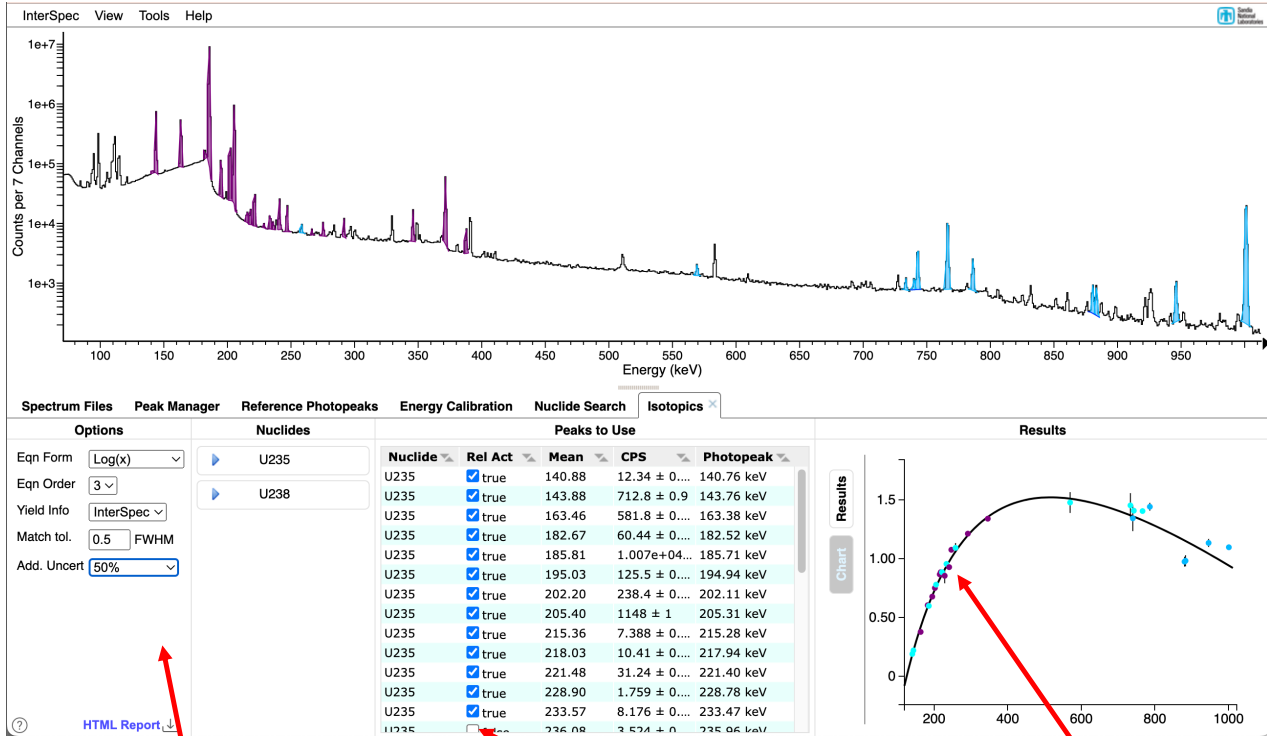


From the "Tools" menu, select:
"Isotopics from peaks"

A new "Isotopics" tab will appear



Isotopics from peaks: Overview of using the tool



Results

Relative activities and mass fractions

Nuclide	Rel. Act.	Mass Frac.	Uncert.
U235	3.376E7	51.82%	26.4%
U238	4882151	48.17%	25.6%

Mass and Activity Ratios.

Nuclides	Mass Ratio	Activity Ratio
U238/U235	0.9296 ± 0.0108	0.1446 ± 0.0697
U235/U238	1.076 ± 21.42	6.915 ± 3.332

Rel. Eff.: $y = 23.173 - 17.537 \cdot \ln(x)^1 + 3.95 \cdot \ln(x)^2 - 0.27174 \cdot \ln(x)^3$

$\chi^2=16226$ and there were 23 DOF ($\chi^2/DOF=705.5$)

**Uncertainties artificially increased by "Add. Uncert".
The tool is in an alpha-preview state only.**

Fiddle with settings until Rel Eff chart looks good

Select which peaks you want, until Rel Eff chart looks okay

You want this chart/fit to look "good"

Take a look at text results

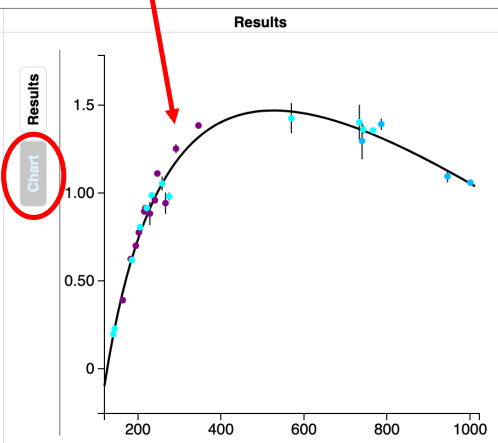
Isotopics from peaks: parts of the tool



Calculation Options

The current Relative Efficiency curve fit
 Fit is automatically updated as you change settings, add/remove peaks, etc
 Your goal is to make this fit to look good!

Nuclide	Rel Act	Mean	CPS	Photopeak
U235	<input checked="" type="checkbox"/>	202.20	238.4 ± 0....	202.11 keV
U235	<input checked="" type="checkbox"/>	205.40	1148 ± 1	205.31 keV
U235	<input checked="" type="checkbox"/>	215.36	7.388 ± 0....	215.28 keV
U235	<input checked="" type="checkbox"/>	218.03	10.41 ± 0....	217.94 keV
U235	<input checked="" type="checkbox"/>	221.48	31.24 ± 0....	221.40 keV
U235	<input checked="" type="checkbox"/>	228.90	1.759 ± 0....	228.78 keV
U235	<input checked="" type="checkbox"/>	233.57	8.176 ± 0....	233.47 keV
U235	<input type="checkbox"/>	236.08	3.524 ± 0....	235.96 keV
U235	<input checked="" type="checkbox"/>	240.96	20.47 ± 0....	240.88 keV
U235	<input checked="" type="checkbox"/>	246.93	16.76 ± 0....	246.84 keV
U238	<input checked="" type="checkbox"/>	258.33	3.384 ± 0....	258.26 keV
U235	<input checked="" type="checkbox"/>	266.51	1.609 ± 0....	266.45 keV
U235	<input checked="" type="checkbox"/>	275.34	5.593 ± 0....	275.13 keV
U235	<input checked="" type="checkbox"/>	291.76	7.128 ± 0....	291.65 keV
U235	<input checked="" type="checkbox"/>	345.94	14.96 ± 0....	345.90 keV
U235	<input type="checkbox"/>	371.47	69.42 ± 0....	371.20 keV
U235	<input type="checkbox"/>	387.87	6.554 ± 0....	387.82 keV



The text results of fit
 (the "truth" enrichment for this data is 52.49%)

Results

Warning: Additional uncertainties were applied to peaks - the result uncertainties include these, so may not be reliable to interpret. The χ^2/DOF does not include the add. uncerts.

Relative activities and mass fractions			
Nuclide	Rel. Act.	Mass Frac.	Uncert.
U235	3.432E7	52.26%	26.1%
U238	4876686	47.74%	26.8%

Mass and Activity Ratios		
Nuclides	Mass Ratio	Activity Ratio
U238/U235	0.9133 ± 0.0107	0.1421 ± 0.0691
U235/U238	1.095 ± 21.99	7.038 ± 3.421

Rel. Eff.: $y = 19.117 - 15.139 \ln(x)^*1 + 3.4803 \ln(x)^*2 - 0.24129 \ln(x)^*3$

$\chi^2=17793$ and there were 20 DOF ($\chi^2/DOF=889.6$)

Export a HTML summary

The nuclides for peaks you have selected to use
 Basic nuclide info is listed
 You can set ages for nuclides that change as they age

The peaks you have fit in the spectrum
 You can choose to use or not-use each peak for the Rel. Eff. analysis
 You can also directly edit nuclide info in this table

Options: Eqn Form

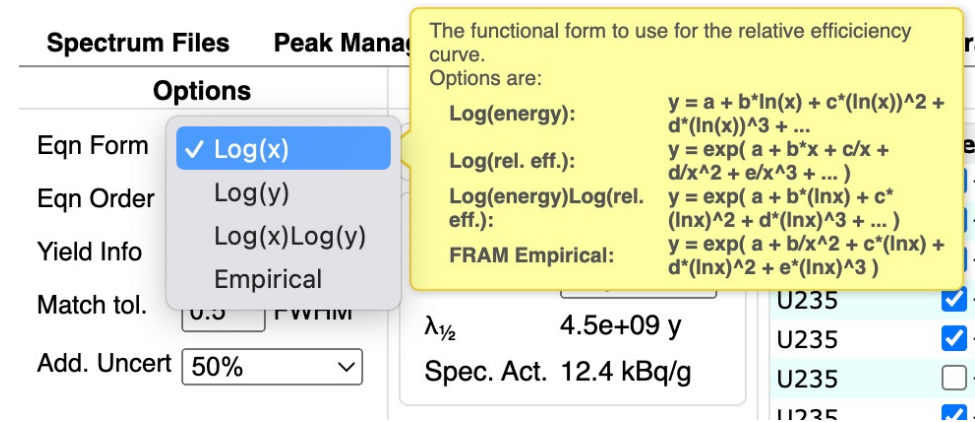
There are a few different functional forms of the Relative Efficiency curve you can choose from:

$$y(x) = c_0 + c_1 \ln^1(x) + c_2 \ln^2(x) + c_3 \ln^3(x) + \dots$$

$$y(x) = \exp\left(c_0 + c_1 x + \frac{c_2}{x} + \frac{c_3}{x^2} + \frac{c_4}{x^3} + \dots\right)$$

$$y(x) = \exp\left(c_0 + c_1 \ln^1(x) + c_2 \ln^2(x) + \dots\right)$$

$$y(x) = \exp\left(c_0 + \frac{c_1}{x^2} + c_2 \ln^1(x) + c_3 \ln^2(x) + \dots\right)$$



The functional form to use for the relative efficiency curve.
Options are:

- Log(energy):** $y = a + b \cdot \ln(x) + c \cdot (\ln(x))^2 + d \cdot (\ln(x))^3 + \dots$
- Log(rel. eff.):** $y = \exp\left(a + b \cdot x + \frac{c}{x} + \frac{d}{x^2} + \frac{e}{x^3} + \dots\right)$
- Log(energy)Log(rel. eff.):** $y = \exp\left(a + b \cdot (\ln(x)) + c \cdot (\ln(x))^2 + d \cdot (\ln(x))^3 + \dots\right)$
- FRAM Empirical:** $y = \exp\left(a + \frac{b}{x^2} + c \cdot (\ln(x)) + d \cdot (\ln(x))^2 + e \cdot (\ln(x))^3\right)$

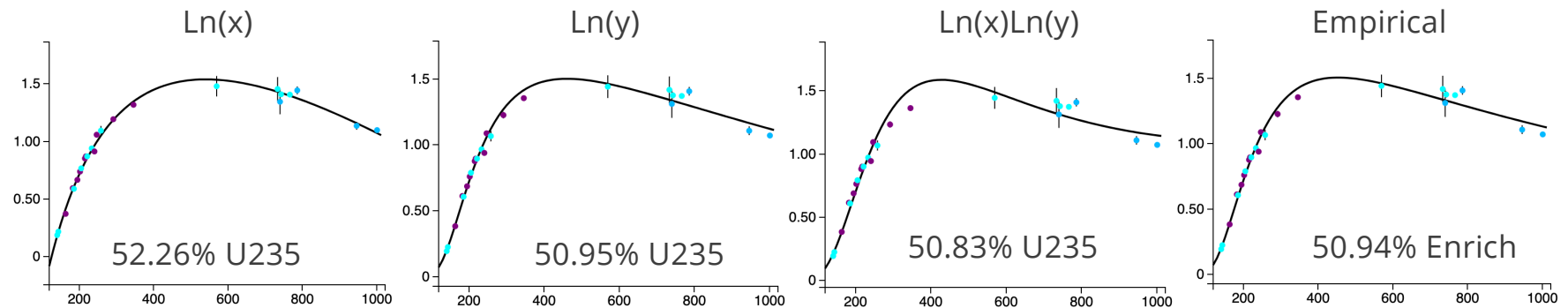
Parameter	Value	U235	U235
$\lambda_{1/2}$	4.5e+09 y	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
Spec. Act.	12.4 kBq/g	<input type="checkbox"/>	<input checked="" type="checkbox"/>

Enable "Help" → "Options" → "Show tooltips"
to see descriptions like above yellow box

Options: Eqn Form (cont)



Generally you want to choose the *Eqn Form* that *looks* the best.

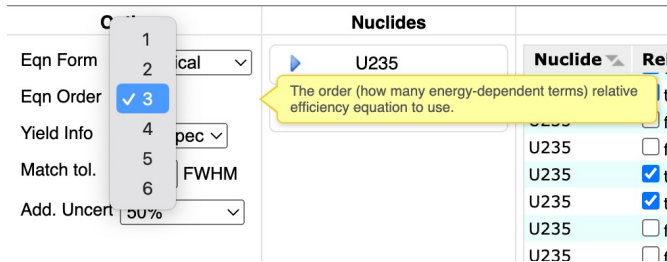


(all fits use 3 energy dependent terms)

Subjectively looks best
- data *looks* to follow line best

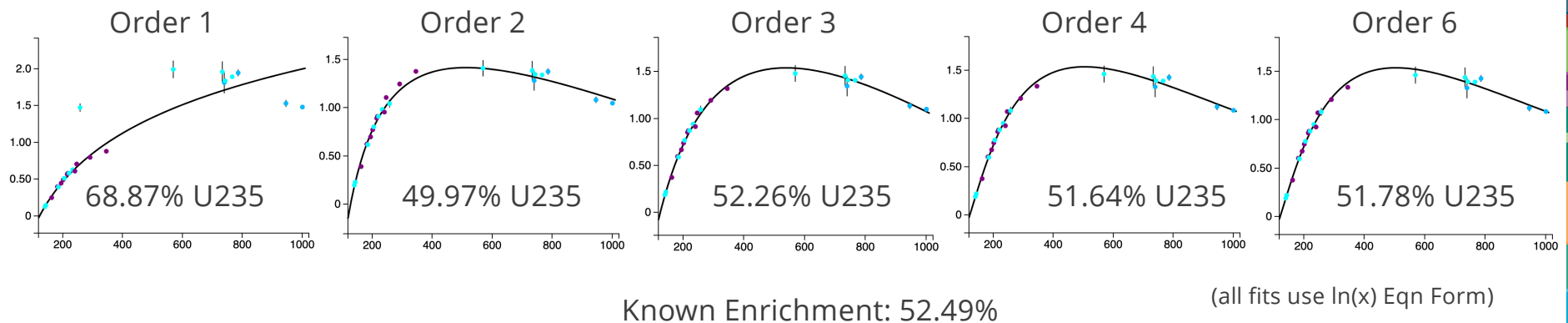
Known Enrichment: 52.49%

Options: Eqn Order

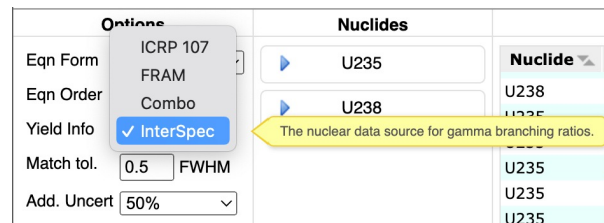


Eqn Order is the number of energy dependent terms in the Relative Efficiency equation.

Generally: use the fewest number of terms that make the Relative Efficiency curve fit *look* good. Too many terms, especially with smaller number of peaks, can allow the results to be non-physical.



Options: Yield Info



This option only appears for problems with Uranium in them.

It allows selecting the Branching Ratio source data for Uranium.

ICRP 107, FRAM, and "Combo" are limited to the major "clean" uranium lines (i.e., not all your peaks may be useable)

"Combo" is a combination of ICRP 107, FRAM, and Sandia branching ratios, that seems to work well

Recommend: use "InterSpec"

Options: Match tol.

Options		Nuclides	
Eqn Form	Log(x) ▾	▶ U235	Nuclide ▾ Re
Eqn Order	3 ▾		
Yield Info	InterSpec ▾		
Match tol.	0.5 FWHM		
Add. Uncert	50% ▾		

The number of FWHM, from the peak mean, to include source gammas from as contributing to a peaks area. For some photopeaks of some nuclides multiple gammas that are close in energy may contribute to creating a detected peak area. This match tolerance specifies how many FWHM from the observed peak mean source gammas should be summed to determine the branching ratio to use. Specifying a value of zero will cause only the gamma energy assigned to a peak to be used, even if there are very nearby other gammas.

This option allows InterSpec to correctly account for multiple gammas, maybe from multiple nuclides, contributing to a single observable peak.

The gamma (i.e., the nuclide and specific energy gamma) you associate with a peak will always be used in the relative efficiency calculation.

However, the fit peak may have contributions from nearby gammas of either the same nuclide, or any of the other nuclides in the problem.

The “Match tol.” option defines how many full-width-at-half-maximum a nuclides gamma, from any of the nuclides being used for the Rel. Eff. calculation, can be away from the peak mean, and still considered to contribute to the peak.

A value of “0” makes it so only the gamma you assigned to the peak will be used.

Options: Add. Uncert

Recommendation: leave this options as “Stat. Only”, except for Uranium problems, then use 50%

An additional uncertainty to add to the relative efficiency line, for each fit photopeak. Small deviations in efficiency of detection for one or a few high statistics peaks can cause the efficiency curve to notably deviate from the other points if only statistical uncertainties are used; when an additional uncertainty is added the relative efficiency will then do a better job of visibly going through all the data points, and from limited testing produce more accurate results. You can think of this as adding a systematic uncertainty to each detected photopeak, that is uncorrelated between peaks. From limited testing the value used is not hugely important, just as long as there is something. You can also choose to use an unweighted fit, where each peak will contribute to the fit equally, no matter its statistical uncertainty.

Eqn Form	Eqn Order	Match tol.	Add. Uncer
Log(x)	4	Unweighted	Stat. Only
		1%	
		5%	
		10%	
		25%	

This option is useful for high-statistics samples where systematic errors in fitting peak amplitude (e.g., imperfect continuum or skew modeling) are larger than statistical uncertainties of the fit.

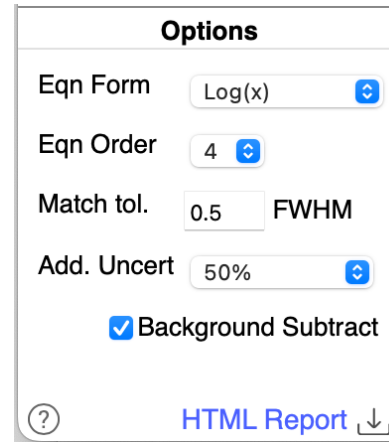
For high-statistics peaks, fitting an amplitude that is only a very tiny percentage off can be many, many statistical sigma off, so a peak may disproportionately impact the Relative Efficiency curve fit (e.g., cause a “kink”, or discontinuity in it).

This option allows accounting for this effect – at the cost of making the computed uncertainties no longer easily interpretable

In limited testing, this option is mostly only **useful for Uranium problems**.

- For Uranium problems, from a limited test set, adding an additional uncertainty increased the accuracy of the computed answer, in comparison to known values
- The value used (e.g., 1%, 5%, 10%, etc) doesn't seem to matter much, just as long as *some* additional uncertainty is used – but using 50% did perform slightly better than other values

Options: Background Subtract



Options

Eqn Form

Eqn Order

Match tol. FWHM

Add. Uncert

Background Subtract

[HTML Report](#) ↓

This option subtracts background peak areas from foreground peak areas.

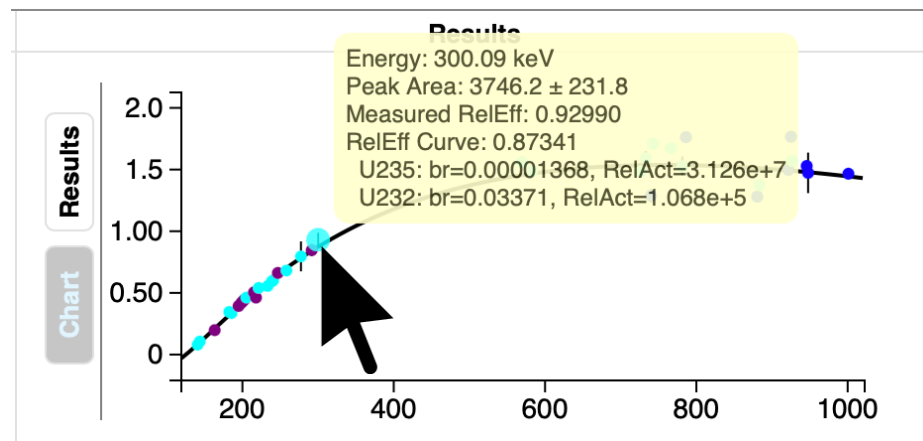
You must first load the background spectrum as a foreground and fit the relevant peaks in it (nuclide associations do not need to be made), then use the "Spectrum Files" tab to change the spectrum to the background, then load/select spectrum of interest as the foreground spectrum

This is not a channel-by-channel subtraction, but instead it matches peaks in the foreground to peaks already fit in the background, and subtracts their area and variances

- A matching tolerance of about 0.5 FWHM between foreground and background peaks is used

This option is only shown if there is a background spectrum file loaded, with peaks fit.

Additional features: Rel. Eff. Chart



“Mousing” over data points on Relative Efficiency chart will show a box that gives further information about that data point.

Additional features: Nuclide Age

Nuclides	
▶	U232
▶	U235
▼	U238
Age	<input type="text" value="20 y"/>
$\lambda_{1/2}$	4.5e+09 y
Spec. Act.	12.4 kBq/g

You can set a nuclide's age by expanding its entry in the "Nuclides" column



Additional features: HTML Report

Options

Eqn Form

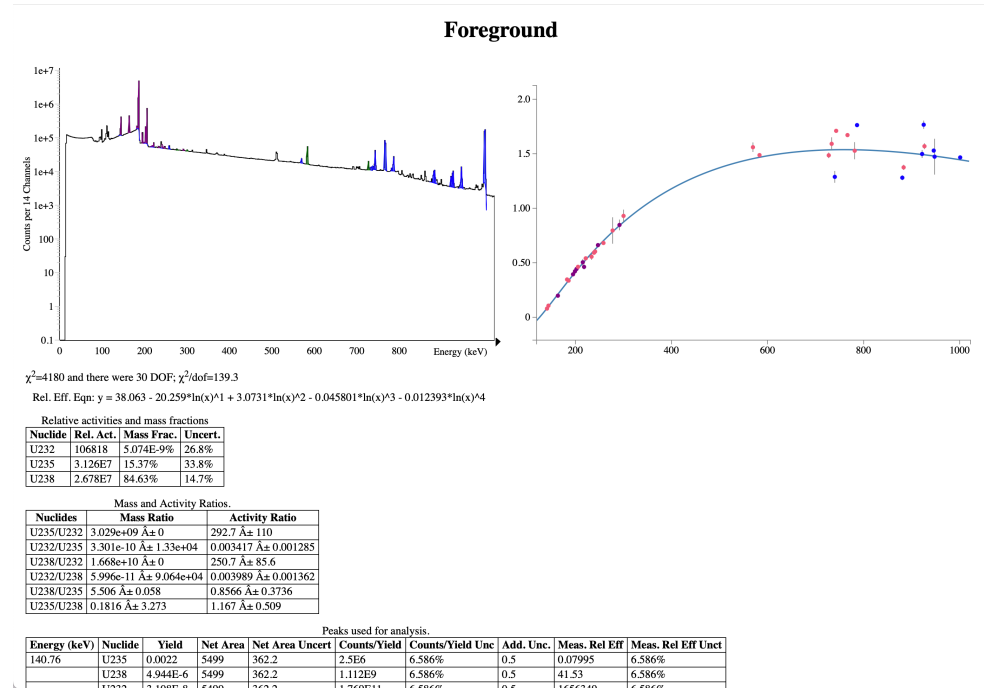
Eqn Order

Yield Info

Match tol. FWHM

Add. Uncert

[HTML Report](#) ↓



You can export a HTML report that contains an interactive spectrum and relative efficiency charts, as well as additional information not in the InterSpec GUI.

Additional Uses of the Tool



Although primarily intended for determining the ratio of nuclide activities and masses, the tool can also be useful for:

- Checking for unexpected interferences, or other effects.
E.x., If a peak is substantially away from the fit relative efficiency curve, then it is possible another unknown nuclide is contributing to that peak.
- If two peaks are close together in energy, then their attenuation and detection efficiencies will be about the same, so you can use a order 0 equation to estimate their relative activities, without using the full energy range
- You can get a nuclides age, by manually changing the age entry until the relative Efficiency fit looks good